TRIGONOMETRY

Trigonometric Ratios

Example 1

Given the right-angled triangle ABC and that $\tan \theta = 2$, find $\sin \theta$ and $\cos \theta$.

Trigonometric Ratios of Some Special Angles

The trigonometric ratios of angles measuring $30^\circ$, $45^\circ$ and $60^\circ$ can be obtained using the right-angled triangles formed by taking ‘half’ of a square and an equilateral triangle.

<table>
<thead>
<tr>
<th>Trig / $\theta$</th>
<th>$0^\circ$</th>
<th>$30^\circ$</th>
<th>$45^\circ$</th>
<th>$60^\circ$</th>
<th>$90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cos</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sin</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tan</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Trigonometric Ratios of Complementary Angles

In the triangle PQR in the diagram, $\angle P$ and $\angle R$ are complementary angles as $\angle P + \angle R = 90^\circ$.

Example 2

Deduce the value of \[
\frac{\cos 20^\circ}{\cos 20^\circ + \sin 70^\circ}
\]
General Angles

In general, the angle $\theta$ is measured from the positive $x$-axis.

(a) $\theta$ is positive if it is measured in an anti-clockwise direction.

(b) $\theta$ is negative if it is measured in a clockwise direction.

(c) For positive $\theta$, the quadrant of $\theta$ is determined by the value of $\theta$.

The following four diagrams show some values of $\theta$ in the four quadrants, with a basic angle $\alpha = 45^\circ$. Observe that all the angle $\theta$ are in the **anticlockwise** sense as indicated by the arrowheads and the values of $\theta$ are said to be positive.

In the following four diagrams, $\alpha = 45^\circ$. Observe that the angles $\theta$ are negative in value because they are in the **clockwise** sense as indicated by the arrowheads.
Example 3

Given that $0^\circ < \theta < 360^\circ$ and $\theta$ has a basic angle of $40^\circ$, find $\theta$ if it is in the
(a) 3$^{rd}$ quadrant  (b) 4$^{th}$ quadrant

Basic Angle $\alpha$

$\alpha$ is an acute angle. It is made by the arm of the line segment and the x-axis. The sine, cosine and tangent of a basic angle should be taken as positive. Below are the position of basic angle $\alpha$ in the four quadrant:

![Diagram of the four quadrants with basic angles]

Signs of Trigonometric Ratios in the Four Quadrants

<table>
<thead>
<tr>
<th></th>
<th>1$^{st}$ Quad</th>
<th>2$^{nd}$ Quad</th>
<th>3$^{rd}$ Quad</th>
<th>4$^{th}$ Quad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sin</td>
<td>+ve</td>
<td>+ve</td>
<td>-ve</td>
<td>-ve</td>
</tr>
<tr>
<td>Cos</td>
<td></td>
<td></td>
<td>-ve</td>
<td>-ve</td>
</tr>
<tr>
<td>Tan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example 4

Without using a calculator, determine whether the following trigonometric ratios are positive or negative.

(a) $\sin 230^\circ$  (b) $\cos 140^\circ$  (c) $\tan 215^\circ$

(d) $\cos 350^\circ$  (e) $\tan 340^\circ$  (f) $\sin 160^\circ$

(g) $\cos (-160^\circ)$  (h) $\tan (-155^\circ)$
Example 5
Evaluate the following (without using calculator):

(a) $\tan 300^\circ$  
(b) $\cos 330^\circ$  
(c) $\sin 150^\circ$  
(d) $\tan 315^\circ$

(e) $\sin 225^\circ$  
(f) $\cos 210^\circ$  
(g) $\tan (-120^\circ)$  
(h) $\sin 405^\circ$

Example 6
Without using a calculator, evaluate the trigonometric ratios of $300^\circ$. Hence deduce the trigonometric ratios of $660^\circ$.

Example 7
Given that $\cos \theta = -\frac{4}{5}$ and $180^\circ < \theta < 270^\circ$, evaluate $\tan \theta$ and $\sin \theta$.

Example 8
Express the trigonometric ratios of $-70^\circ$ in terms of the ratios of its basic angle.

The above results are particular cases of the following more general results:

For any angle $\theta$,

$\cos (-\theta) =$

$\sin (-\theta) =$

$\tan (-\theta) =$
EXERCISE 5A – TRIGONOMETRIC RATIOS AND GENERAL ANGLES

1. For each of the following triangles, find the values of $\cos \theta$, $\sin \theta$ and $\tan \theta$.

   (a) 
   \[
   \begin{array}{c}
   \triangle \quad \theta \\
   5 \quad 4 \quad \theta
   \end{array}
   \]

   (b) 
   \[
   \begin{array}{c}
   \triangle \quad \theta \\
   2 \quad \theta \quad 3
   \end{array}
   \]

   (c) 
   \[
   \begin{array}{c}
   \triangle \quad \theta \\
   13 \quad 5
   \end{array}
   \]

2. For the given right-angled triangle below, complete the table of trigonometric ratios on the right :

<table>
<thead>
<tr>
<th></th>
<th>Cos $\theta$</th>
<th>Sin $\theta$</th>
<th>Tan $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\frac{8}{17}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td>$\frac{7}{25}$</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td></td>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

3. Given that $\sin \theta = \frac{1}{2}$, find the value of $\sin \theta \cos (90^\circ - \theta)$.

4. Given that $\tan A = 2$, find the value of $2 \tan A + \tan (90^\circ - A)$.

5. Without using a calculator, evaluate

   (a) $\frac{\sin 65^\circ}{\cos 30^\circ + \sin 60^\circ}$

   (b) $\tan 45^\circ + \tan 30^\circ \tan 60^\circ$

6. Without using a calculator, find the value of

   (a) $\frac{\sin 65^\circ}{\cos 25^\circ}$

   (b) $\tan 75^\circ \tan 15^\circ$

7. State the quadrant of the angle $\theta$ and find the value of the basic angle $\alpha$ if

   (a) $\theta = 250^\circ$

   (b) $\theta = 390^\circ$

   (c) $\theta = -60^\circ$

   (d) $\theta = -100^\circ$

8. For each of the following diagrams, find the value of the angle $\theta$.

9. Find all the angles between $0^\circ$ and $360^\circ$ which make a basic angle of $\alpha$ with the x-axis if

   (a) $\alpha = 20^\circ$

   (b) $\alpha = 70^\circ$

   (c) $\alpha = 35^\circ$

10. Find all the angles between $-180^\circ$ and $180^\circ$ with basic angle $\alpha = 10^\circ$. 
Example 2: Effect of $\sin bx, \cos bx, \tan bx$

Sketch the graph of the following.

a) $y = \sin 2x$ for $0^\circ \leq x \leq 180^\circ$

b) $y = \cos \frac{1}{2} x$ for $0^\circ \leq x \leq 360^\circ$.

Example 3: Effect of $\sin x + c$, $\cos x + c$, $\tan x + c$

Sketch the graph of the following $0 \leq x \leq 2\pi$.

a) $y = \sin x + 2$
b) \( y = \cos x - 3 \)

**Example 4 : Effect of \( a \sin bx + c, a \cos bx + c, a \tan bx + c \)**

Sketch the graph of the following \( 0^\circ \leq x \leq 360^\circ \).

a) \( y = 2 \sin 3x + 1 \)

b) \( y = 3 \cos 2x - 2 \)

**EXERCISE 5B : SKETCHING TRIGONOMETRIC GRAPHS**

1. Sketch on separate diagram, the following for \( 0^\circ \leq x \leq 360^\circ \) and state the corresponding range of \( y \).
   (a) \( y = \sin 2x - 1 \)    (b) \( y = 4 \sin 3x + 2 \)    (c) \( y = 3 + 4 \cos 2x \)    (d) \( y = 3 \left| \cos 2x \right| \)

2. Sketch on separate diagram, the following for the interval \( 0 \leq x \leq 2\pi \).
   (a) \( y = 5 \cos 2x \)    (b) \( y = 1 + \sin 2x \)    (c) \( 4 \sin 3x - 2 \)    (d) \( 3 \cos 2x - 1 \)
Solving Basic Trigonometric Equations

Basic trigonometric equations of the form \( \cos \theta = \pm k \), \( \sin \theta = \pm k \) and \( \tan \theta = \pm k \), where \( k \) is a constant, are solved as follows:

**Step 1 : Identify the quadrant by looking at the sign of \( k \),**

**Step 2 : Find the basic angle by taking the *positive* value of \( k \) only,**

**Step 3 : Find all the values \( \theta \) and the limit of \( \theta \).**

**Example 1**

Find all the value of \( \theta \) such that

a) \( \sin \theta = -0.5 \) where \( 0^\circ < \theta < 360^\circ \),

b) \( \tan \theta = -1 \) where \( 0^\circ < \theta < 360^\circ \),

c) \( \cos \theta = 0.46 \) where \( -360^\circ < \theta < 0^\circ \).

**Example 2**

Find all the angles between \( 0^\circ \) and \( 360^\circ \) which satisfy the equation

(a) \( \cos (x + 30^\circ) = -0.3 \)  
(b) \( \sin 2x = 0.866 \)

(c) \( \tan (2x - 50^\circ) = -0.7 \)  
(d) \( \cos^2 x = \frac{1}{4} \)

(e) \( \sin^2 x + \sin x \cos x = 0 \)  
(f) \( 6 \cos^2 x - \cos x - 1 = 0 \)

**Example 3**

Solve \( 3(\tan x + 1) = 2 \) for \( -180^\circ \leq x \leq 180^\circ \).

**Example 4**

Solve the following equations for \( 0 \leq x \leq 2\pi \), leaving your answers in terms of \( \pi \).

(a) \( 4 \cos^2 x = 3 \)  
(b) \( 1 - 2 \sin^2 x = 3 \sin x + 2 \)

**EXERCISE 5C : SOLVING BASIC TRIGONOMETRIC EQUATIONS**
1. Find all the angles x where \(0^\circ < x < 360^\circ\) such that
   (a) \(\cos x = -0.71\)  
   (b) \(\tan x = 1.732\)  
   (c) \(\sin x = 0.866\)  
   (d) \(\tan x = -2\)  
   (e) \(10 \cos x - 3 = 0\)  
   (f) \(4(\tan x - 1) = 3(5 - 2 \tan x)\)  
   (g) \(2 \sin(-x) = 0.3\)  
   (h) \(2 \cos^2 x = 1\)  
   (i) \(3 \sin x + 2 = \tan 75^\circ\)  
   (j) \(\frac{8 \cos x + 1}{2 - \cos x} = 3\)  

2. Find all the angles between \(-360^\circ\) and \(180^\circ\) such that
   (a) \(\sin x = -\frac{1}{2}\)  
   (b) \(\cos x = \sqrt{3}/2\)  
   (c) \(\tan(-x) + 1 = 0\)  
   (d) \(\sqrt{2} \sin(90^\circ - 1) + 1 = 0\)  

3. Positive angles x and y are such that \(x + 2y = 300^\circ\) and \(\tan y = 2 \cos 160^\circ\). Find their values.

4. Find the values of x, where \(0 \leq x \leq 2\pi\), which satisfy each of the following equations.
   (a) \(\cos x = \frac{\sqrt{3}}{2}\)  
   (b) \(\tan x = 1\)  
   (c) \(\sin x = -0.5\)  
   (d) \(6 \sin^2 x - 1 = 0\)  
   (e) \(2 \sin x \cos x = \cos x \cos 40^\circ\)  
   (f) \(10 \cos 2x = 7\)  

5. Solve the following equations for all values of x from \(-180^\circ\) to \(+180^\circ\).
   (a) \(\cos (x - 20^\circ) = -\frac{1}{\sqrt{2}}\)  
   (b) \(\cos x (\sin x - 1) = 0\)  
   (c) \(3 \sin^2 x = 2 \sin x \cos x\)  
   (d) \(2 \cos^3 x - 5 \cos x + 2 = 0\)  

**ANSWERS**

1. (a) \(135.2^\circ, 224.8^\circ\)  
   (b) \(60^\circ, 240^\circ\)  
   (c) \(60^\circ, 120^\circ\)  
   (d) \(116.6^\circ, 296.6^\circ\)  
   (e) \(72.5^\circ, 287.5^\circ\)  
   (f) \(62.2^\circ, 242.2^\circ\)  
   (g) \(188.6^\circ, 351.4^\circ\)  
   (h) \(45^\circ, 135^\circ, 225^\circ, 315^\circ\)  
   (i) \(35.3^\circ, 144.7^\circ\)  
   (j) \(63.0^\circ, 297.0^\circ\)  

2. (a) \(-150^\circ, -30^\circ\)  
   (b) \(-330^\circ, -30^\circ, 30^\circ\)  
   (c) \(-315^\circ, -135^\circ, 45^\circ\)  
   (d) \(-225^\circ, -135^\circ, 135^\circ\)  

3. \(x = 64.0^\circ, y = 118.0^\circ\).

4. (a) \(\frac{\pi}{6}, \frac{11\pi}{6}\)  
   (b) \(\frac{\pi}{4}, \frac{5\pi}{4}\)  
   (c) \(\frac{7\pi}{6}, \frac{11\pi}{6}\)  
   (d) \(0.421, 2.72, 3.56, 5.86\)  
   (e) \(\frac{3\pi}{2}, 0.393, 2.75\)  
   (f) \(0.398, 2.74, 3.54, 5.89\)

5. (a) \(-115^\circ, 155^\circ\)  
   (b) \(\pm 90^\circ\)  
   (c) \(0, 33.7^\circ, -146.3^\circ, \pm 180^\circ\)  
   (d) \(\pm 60^\circ\)
Simple Identities

The identities shown are commonly used to:

(a) Solve a trigonometric equation which could be simplified to basic equation(s) of the form \( \sin x = k, \cos x = k \) and \( \tan x = k \).

(b) Solve a trigonometric equation which could be simplified to basic equation(s) of the form \( \sin (ax + b) = k, \cos (ax + b) = k \) and \( \tan (ax + b) = k \).

(c) Prove other identities.

Using Simple Identities To Solve Trigonometric Equations

Example 1

Find all the angles between \( 0^\circ \) and \( 360^\circ \) which satisfy the equation \( 3 \cos x + 2 \sin x = 0 \).

Example 2

Find all the angles between \( 0^\circ \) and \( 360^\circ \) inclusive for which

(a) \( 2 \sin x \cos x = \sin x \)

(b) \( \cos^2 y - \cos y = 2 \)

Example 3

Find all the angles between \( 0 \ll x \ll 2\pi \) which satisfy the equation \( \sin y = 4 \tan y \).

Example 4

Find all the angles between \( 0^\circ \) and \( 360^\circ \) which satisfy the equation \( 2 \cos^2 y - 1 = \sin y \).

Example 5

Find the values of \( x \), where \( 0 \ll x \ll 2\pi \), which satisfy the following equations.

(a) \( 2 \sin x - \cos x = 0 \)

(b) \( 3 \sin x + 2 \cos x = 0 \)
EXERCISE 5D : SOLVING TRIGONOMETRIC EQUATION USING SIMPLE IDENTITIES

1. Find all the angles between 0° and 360° which satisfy the equation
   a) $5 \cos x + 2 \sin x = 0$
   b) $3(\sin x - \cos x) = \cos x$

2. Find all the angles between 0° and 360° inclusive which satisfy the following equations.
   a) $4 \sin x \cos x = \cos x$
   b) $2 \cos^2 x - \cos x = 1$
   c) $3 \sin 2x + 2 = 0$
   d) $2 \sin x \cos x = \tan x$

3. Find all the angles between 0° and 360° which satisfy the following equations.
   a) $\cos 2x = 0.5$
   b) $\tan (x - 60°) = \frac{1}{\sqrt{3}}$
   c) $3 \sin 2x + 2 = 0$
   d) $\cos (2x - 40°) = 0.8$

4. Find all the angles between 0° and 360° which satisfy the equation
   a) $\sin \frac{1}{2} x = \frac{1}{2}$
   b) $\tan (2x - 60°) = -1$
   c) $2 \sin y = \tan y$

5. Solve the following equations for angles between 0° and 360° inclusive.
   a) $2 \cos 2x + 1 = \sqrt{2}$
   b) $2 \cos^2 x + 3 \sin x = 3$
   c) $3 \sin x + 2 \tan x = 0$
   d) $2 \sin x \cos x + \cos^2 x = 1$
   e) $3 \cos (x + 40°) = 4 \sin (x + 40°)$
   f) $2 \sin x \tan x = 3$

ANSWERS :

1. a) 111.8°, 291.8°
   b) 53.1°, 233.1°

2. a) 14.5°, 90°, 165.5°, 270°
   b) 0°, 120°, 240°, 360°
   c) 45°, 108.4°, 225°, 288.4°
   d) 0°, 45°, 135°, 180°, 225°, 315°, 360°

3. a) 30°, 150°, 210°, 330°
   b) 90°, 270°
   c) 110.9°, 159.1°, 290.9°, 339.1°
   d) 1.6°, 38.4°, 181.6°, 218.4°

4. a) 60°, 300°
   b) 7.5°, 97.5°, 187.5°, 277.5°

5. a) 39.0°, 141.0°, 219.0°, 321.0°
   b) 30°, 90°, 150°
   c) 0°, 131.8°, 180°, 228.2°, 360°
   d) 0°, 63.4°, 180°, 243.4°, 360°
   e) 176.9°, 356.9°
   f) 60°, 300°
Proving Identities

\[
\sin^2 \theta + \cos^2 \theta = 1, \\
\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cos \theta \neq 0
\]

The basic identities above may be used to prove other identities as shown in the following examples. In proving trigonometric identities, one usually starts with the more complicated side.

Examples:

Prove the following identities:

1. \[1 - 2 \sin^2 x \equiv 2 \cos^2 x - 1\]
2. \[1 - \frac{\cos^2 x}{1 + \sin x} \equiv \sin x\]
3. \[\sin^4 x - \cos^4 x \equiv \sin^2 x - \cos^2 x\]
4. \[\frac{1}{\tan \theta} + \tan \theta \equiv \frac{1}{\sin \theta \cos \theta}\]
**EXERCISE 5E : PROVING IDENTITIES**

1. Prove the following identities:

   a) \[ \sin x \tan x \equiv \frac{1 - \cos^2 x}{\cos x} \]
   
   b) \[ (1 - \cos x)(1 + \frac{1}{\cos x}) \equiv \sin x \tan x \]

   c) \[ \frac{1 + \sin x}{\cos x} \equiv \frac{\cos x}{1 - \sin x} \]

   d) \[ \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \equiv \frac{2}{\cos^2 \theta} \]

   e) \[ (\tan \theta + \sin \theta)(\tan \theta - \sin \theta) \equiv \tan^2 \theta \sin^2 \theta \]

   f) \[ (1 + \sin \theta)(\frac{1}{\cos \theta} - \tan \theta) \equiv \cos \theta \]

   g) \[ \sin^2 A + \tan^2 A \sin^2 A \equiv \tan^2 A \]

   h) \[ \frac{1}{\sin \theta} - \sin \theta \equiv \frac{\cos \theta}{\tan \theta} \]

2. Show that the equation \(15 \cos^2 \theta = 13 + \sin \theta\) may be written as a quadratic equation in \(\sin \theta\).

3. It is given that \(a = 2 \sin \theta + \cos \theta\) and \(b = 2 \cos \theta - \sin \theta\). Show that \(a^2 + b^2\) is constant for values of \(\theta\).
1. Show that the equation $15 \cos^2 \theta = 13 + \sin \theta$ may be written as a quadratic equation in $\sin \theta$. Hence solve the equation, for $0^\circ \leq \theta \leq 360^\circ$. Give your answers correct to the nearest $0.1^\circ$. [Specimen paper Q5] [2] [5]

2. Prove the identity $(1 + \sin \theta)(\frac{1}{\cos \theta} - \tan \theta) \equiv \cos \theta$. [June 2001 Q4] [4]

3. (i) Sketch and label, on the same diagram, the graphs of $y = \cos x$ and $y = \cos 3x$ for the interval $0 \leq x \leq 2\pi$. [2]

   (ii) Given that $f : x \rightarrow \cos x$, for the domain interval $0 \leq x \leq k$, find the largest value of $k$ for which $f$ has an inverse. [2]

4. It is given that $a = 2 \sin \theta + \cos \theta$ and $b = 2 \cos \theta - \sin \theta$, where $0^\circ \leq \theta \leq 360^\circ$.
   (i) Show that $a^2 + b^2$ is constant for all value of $\theta$. [3]
   (ii) Given that $2x = 3b$, show that $\tan \theta = 4/7$ and find the corresponding values of $\theta$. [4]

5. (i) Show that $\sin x \tan x$ may be written as $\frac{1-\cos^2 x}{\cos x}$. [1]

   (ii) Hence solve the equation $2 \sin x \tan x = 3$, for $0^\circ \leq \theta \leq 360^\circ$. [4]

6. (i) Show that the equation $3 \tan \theta = 2 \cos \theta$ can be expressed as $2 \sin^2 \theta + 3 \sin \theta - 2 = 0$. [3]

   (ii) Hence solve the equation $3 \tan \theta = 2 \cos \theta$, for $0^\circ \leq \theta \leq 360^\circ$. [3]

7. In the diagram, triangle ABC is right-angled and D is the mid-point of BC. Angle DAC = $30^\circ$ and angle BAD = $x^\circ$. Denoting the length of AD by $l$,

   (i) express each of AC and BC exactly in terms of $l$, and show that $AB = \frac{1}{2} l\sqrt{7}$. [4]

   (ii) show that $x = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) - 30$. [Nov 2002 Q6]

8. Find the values of $x$ in the interval $0^\circ \leq \theta \leq 180^\circ$ which satisfy the equation $\sin 3x + 2 \cos 3x = 0$. [June 2003 Q2] [4]

9. (i) Sketch the graph of the curve $y = 3 \sin x$, for $-\pi \leq x \leq \pi$. The straight line $y = kx$, where $k$ is a constant, passes through the maximum point of this curve for $-\pi \leq x \leq \pi$. [June 2003 Q6] [2]

   (ii) Find the value of $k$ in terms of $\pi$. [2]

   (iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]
10. (i) Show that the equation \(4 \sin^2 \theta + 5 = 7 \cos^2 \theta\) may be written in the form \(4x^2 + 7x - 2 = 0\), where \(x = \sin^2 \theta\). \[1\]

(ii) Hence solve the equation \(4 \sin^4 \theta + 5 = 7 \cos^2 \theta\), for \(0^\circ \leq \theta \leq 360^\circ\). \[4\] [Nov 2003 Q 2]

11. (i) Show that the equation \(\sin^2 \theta + 3 \sin \theta \cos \theta = 4 \cos^2 \theta\) can be written as a quadratic equation in \(\tan \theta\). \[2\]

(ii) Hence, or otherwise, solve the equation in part (i) for \(0^\circ \leq \theta \leq 180^\circ\). \[3\] [June 2004 Q3]

12. (i) Sketch and label, on the same diagram, the graphs of \(y = 2 \sin x\) and \(y = \cos 2x\), for the interval \(-\pi \leq x \leq \pi\). \[4\]

(ii) Hence state the number of solutions of the equation \(2 \sin x = \cos 2x\) in the interval \(0 \leq x \leq \pi\). \[1\] [Nov 2004 Q4]

13. The function \(f : x \rightarrow 5 \sin 2x + 3 \cos 2x\) is defined for the domain \(0 \leq x \leq \pi\).
   (i) Express \(f(x)\) in the form \(a + b \sin 2x\), stating the values of \(a\) and \(b\). \[2\]

(ii) Hence find the values of \(x\) for which \(f(x) = 7 \sin x\). \[3\]

(iii) State the range of \(f\). \[2\] [Nov 2004 Q6]

14. (i) Show that the equation \(\sin \theta + \cos \theta = 2(\sin \theta - \cos \theta)\) can be expressed as \(\tan \theta = 3\). \[2\]

(ii) Hence solve the equation \(\sin \theta + \cos \theta = 2(\sin \theta - \cos \theta)\), for \(0^\circ \leq \theta \leq 360^\circ\). \[2\] [June 2005 Q3]

15. A function \(f\) is defined by \(f : x \rightarrow 3 - 2 \sin x\), for \(0^\circ \leq \theta \leq 360^\circ\).
   (i) Find the range of \(f\). \[2\]

(ii) Sketch the graph of \(y = f(x)\). \[2\] [June 2005 Q7]

16. Solve the equation \(3 \sin 2\theta - 2 \cos \theta - 3 = 0\), for \(0^\circ \leq \theta \leq 360^\circ\). \[4\] [Nov 2005 Q1]

17. In the diagram, ABED is a trapezium with right angles at E and D, and CED is a straight line. The lengths of AB and BC are \(2d\) and \((2\sqrt{3})d\) respectively, and angles BAD and CBE are \(30^\circ\) and \(60^\circ\) respectively.

   (i) Find the length of CD in terms of \(d\). \[2\]

(ii) Show that angle CAD = \(\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\). \[3\] [Nov 2005 Q 3]

18. Solve the equation \(\sin 2x + 3 \cos 2x = 0\) for \(0^\circ \leq \theta \leq 180^\circ\). \[4\] [June 2006 Q 2]
19. Given that \( x = \sin^{-1} \left( \frac{2}{5} \right) \), find the exact value of
   (i) \( \cos^2x \) 
   (ii) \( \tan^2x \) 
   [Nov 2006 Q 2]  

20. Prove the identity \( \frac{1-\tan^2x}{1+\tan^2x} \equiv 1 - 2\sin^2x \).  
   [Jun 2007 Q3]  

21. (i) Show that the equation \( 3 \sin x \tan x = 8 \) can be written as \( 3 \cos^2x + 8 \cos x = 0 \).  
   (ii) Hence solve the equation \( 3 \sin x \tan x = 8 \) for \( 0^\circ \leq \theta \leq 360^\circ \).  
   [Nov 2007 Q5]  

22. (i) Show that the equation \( 2 \tan^2\theta \cos \theta = 3 \) can be written as \( 3 \cos^2x + 8 \cos x = 0 \).  
   (ii) Hence solve the equation \( 2 \tan^2\theta \cos \theta = 3 \) for \( 0^\circ \leq \theta \leq 360^\circ \).  
   [June 2008 Q2]  

23. Prove the identity \( \frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \equiv \frac{2}{\cos x} \).  
   [Nov 2008 Q2]  

24. Prove the identity \( \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2\tan^2x \).  
   [June 2009 Q1]  

25.  

The diagram shows the graph of \( y = a \sin (bx) + cf \) for \( 0 \leq x \leq 2\pi \).
   (i) Find the values of \( a, b \) and \( c \).  
   (ii) Find the smallest value of \( x \) in the interval \( 0 \leq x \leq 2\pi \) for which \( y = 0 \).  
   [June 2009 Q4]  

26. (i) Prove the identity \( (\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3x + \cos^3x \).  
   (ii) Solve the equation \( (\sin x + \cos x)(1 - \sin x \cos x) \equiv 9 \sin^3x \) for \( 0^\circ \leq \theta \leq 360^\circ \).  
   [Nov 2009 Q5]